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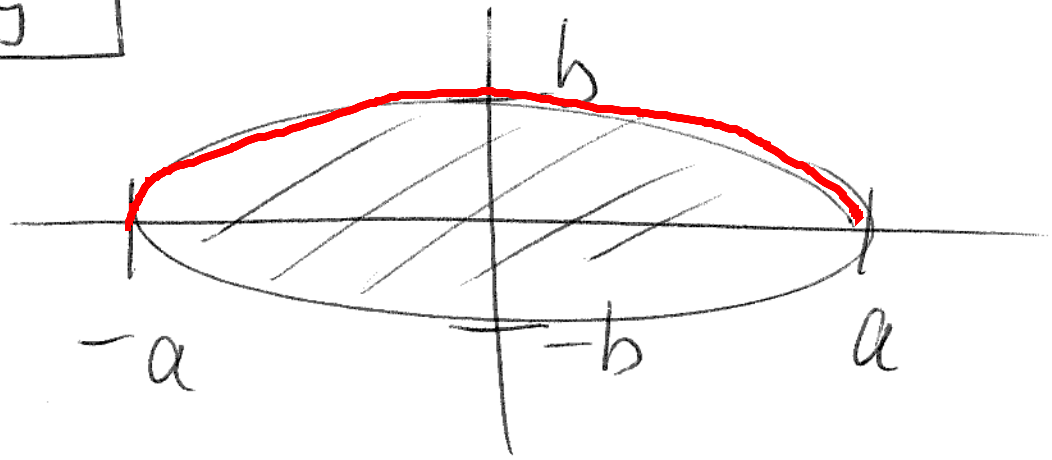

Beh.: $A_{Ell} = \pi \cdot ab$ mit

Relation

$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ansatz

$$A_{Ellipse} = 2 \cdot \int_{-a}^{a} y_{Ell} = \dots \, dx$$

$y = f(x)$ Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Big| - \frac{x^2}{a^2}$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad \Big| \cdot b^2$$

$$y^2 = b^2 - \frac{b^2}{a^2}x^2 \quad \Big| +\sqrt{} > 0$$

$$y = +\sqrt{b^2 - \frac{b^2}{a^2}x^2}$$

Umformungen [muß nicht sein!!]

$$\frac{a^2b^2 - b^2x^2}{a^2} \qquad y = \sqrt{\frac{a^2b^2}{a^2} - \frac{b^2}{a^2}x^2}$$

$$\frac{b^2}{a^2}\left[a^2 - x^2\right] \qquad y = \sqrt{\frac{b^2}{a^2} \cdot (a^2 - x^2)}$$

$$\boxed{y = \frac{b}{a} \cdot \sqrt{a^2 - x^2}}$$

Wir berechnen eine Stammfunktion

$$\int \frac{b}{a}\sqrt{a^2-x^2} = \frac{b}{a}\cdot \int \sqrt{a^2-x^2}\,dx$$

Substitution: $x = a \cdot \sin z$

$$x^2 = a^2 \cdot \sin^2(z)$$

Berechnung von dx

$$x = a \cdot \sin(z) \quad \Big| \; \frac{'}{dx}$$

$$1 = z' \cdot a \cdot \sin z'$$

$$1 = \frac{dz}{dx}\, a \, \cancel{\sin z}\,\cos z \quad \Big| \cdot dx$$

$$dx = a \cdot \cancel{\sin z}\,\cos z \, dz$$

$$\frac{b}{a}\cdot \int \sqrt{a^2-x^2}\,dx = \frac{b}{a}\cdot \int \sqrt{a^2 - a^2 \sin^2}\; a \cdot \cancel{\sin z}\,\cos z \, dz$$

$\sin^2 + \cos^2 = 1$

$$= \frac{b}{a}\cdot \int \sqrt{a^2 \cdot (1-\sin^2)}\; a \cos \, dz$$

$$= \frac{b}{a}\cdot a^2 \cdot \int \cos^2 dz$$

$$\int \cos^2(z)\,dz \stackrel{!}{=} \frac{1}{2}\cdot\left[\sin z\cos z + z\right]$$

letztes Video!
Anhang!

$$\Rightarrow = b\cdot a\cdot\frac{1}{2}\left[\sin z\cos z + z\right]$$

Rücksubstitution $x = a\cdot\sin z$

$$\sin z = \frac{x}{a}$$

$$z = \arcsin\left(\frac{x}{a}\right)$$

$$\cos z = \sqrt{1-\sin^2 z}$$

$$= \sqrt{1-\frac{x^2}{a^2}}$$

$$= \sqrt{\frac{a^2-x^2}{a^2}}$$

$$= \frac{1}{a}\sqrt{a^2-x^2}$$

$$\Rightarrow = \frac{1}{2}ab\cdot\left[\frac{x}{a}\cdot\frac{1}{a}\sqrt{a^2-x^2} + \arcsin\left(\frac{x}{a}\right)\right] + C$$

$$A_{Ell} = 2 \cdot \int_{-a}^{a} y_{Ell} \, dx$$

$$= 2 \cdot \left[\frac{1}{2} ab \left\{ \frac{x}{a^2} \sqrt{a^2 - x^2} + \arcsin\left(\frac{x}{a}\right) \right\} \right]_{-a}^{a}$$

$$= 2 \cdot \Bigg\{ \underbrace{\left[\frac{1}{2} ab \left\{ \cancel{\frac{a}{a^2} \sqrt{a^2 - a^2}} + \underbrace{\arcsin(1)}_{\frac{\pi}{2}} \right\} \right]}_{\text{obere Grenze}} - \underbrace{\left[\frac{1}{2} ab \left\{ \cancel{\frac{-a}{a^2} \sqrt{a^2 - a^2}} + \underbrace{\arcsin(-1)}_{-\frac{\pi}{2}} \right\} \right]}_{\text{untere Grenze}} \Bigg\}$$

$$= 2 \cdot \Bigg\{ \underbrace{\frac{1}{2} ab \frac{\pi}{2}}_{\text{o. G.}} + \underbrace{\frac{1}{2} ab \frac{\pi}{2}}_{\text{u. G}} \Bigg\}$$

$$= 2 \cdot \left\{ \frac{\pi ab}{4} + \frac{\pi ab}{4} \right\} = 2 \cdot \frac{\pi ab}{2} = \pi ab$$

Anhang

$$\int \cos^2 = \int \underset{u'\cdot v}{\cos\cdot\cos} = \underset{u\cdot v}{\sin\cdot\cos} - \underbrace{\int \underset{u\cdot v'}{\sin\cdot(-\sin)}}_{\text{Restintegral}}$$

$$= \sin\cdot\cos + \int \sin^2$$

$$= \sin\cdot\cos + \int 1-\cos^2$$

$$= \sin\cdot\cos + \int 1 - \int\cos^2$$

$$= \sin\cdot\cos + x - \int\cos^2$$

also

$$\int\cos^2 = \sin\cdot\cos + x - \int\cos^2 \quad + \int\cos^2$$

$$2\int\cos^2 = \sin\cdot\cos + x \quad | :2$$

$$\int\cos^2(x)\,dx = \frac{1}{2}\cdot\left[\sin(x)\cdot\cos(x) + x\right] + C$$

~~Fragen? Wünsche? Anregungen?~~

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Anhang

$$\int \cos^2 = \int \underset{u' \cdot v}{\cos \cdot \cos} = \underset{u \cdot v}{\sin \cdot \cos} - \int \underset{u \cdot v'}{\sin \cdot (-\sin)}$$

$$= \sin \cdot \cos + \int \sin^2$$

$$= \sin \cdot \cos + \int 1 - \cos^2$$

$$= \sin \cdot \cos + \int 1 - \int \cos^2$$

$$= \sin \cdot \cos + x - \int \cos^2$$

also

$$\int \cos^2 = \sin \cdot \cos + x - \int \cos^2 \quad \Big| + \int \cos^2$$

$$2 \int \cos^2 = \sin \cdot \cos + x \quad \Big| : 2$$

$$\int \cos^2(x)\,dx = \frac{1}{2} \cdot \Big[\sin(x) \cdot \cos(x) + x \Big] + C$$

Fragen? Wünsche? Anregungen?

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